What do teachers do in their classrooms to cause these effects on students?

Cause	Effect
	1. Make sense of problems and persevere in solving them. **DO STUDENTS:** • Use multiple representations (verbal descriptions, symbolic, tables, graphs, etc.)? • Check their answers using different methods? • Continually ask “Does this make sense?” • Understand the approaches of others and identify correspondences between different approaches?

Cause	Effect
	3.Construct viable arguments and critique the reasoning of others. **DO STUDENTS:** • Make conjectures and build a logical progression of statements to explore the truth of their conjectures? • Analyze situations and recognize and use counter examples? • Justify their conclusions, communicate them to others, and respond to arguments of others? • Hear or read arguments of others and decide whether they make sense, and ask useful questions to clarify or improve the argument?

What do teachers do in their classrooms to cause these effects on students?

<table>
<tr><th>Cause</th><th>Effect</th></tr>
<tr><td></td><td>5.Use appropriate tools strategically.
DO STUDENTS:
Consider the available tools when solving mathematical problems?
Know the tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful?
Identify relevant external mathematical resources and use them to pose or solve problems?
Use technological tools to explore and deepen their understanding of concepts?</td></tr>
</table>

<table>
<tr><th>Cause</th><th>Effect</th></tr>
<tr><td></td><td>6.Attend to precision.
DO STUDENTS:
Communicate precisely to others?
Use clear definitions?
Use the equal sign consistently and appropriately?
Calculate accurately and efficiently?</td></tr>
</table>

What do teachers do in their classrooms to cause these effects on students?

Teacher Actions (Cause)	Student Practice (Effect)
	4. Model with mathematics. **DO STUDENTS:** • Apply the mathematics they know to solve problems in everyday life? • Apply what they know and make assumptions and approximations to simplify a complicated situation as an initial approach? • Identify important quantities in a practical situation? • Analyze relationships mathematically to draw conclusions? • Interpret their mathematical results in the context of the situation and reflect on whether the results make sense?

Teacher Actions (Cause)	Student Practice (Effect)
	2.Reason abstractly and quantitatively. • **DO STUDENTS:** • Make sense of quantities and their relationships in problem situations? • Decontextualize a problem? • Contextualize a problem? • Create a coherent representation of the problem, consider the units involved, and attend to the meaning of quantities?

What do teachers do in their classrooms to cause these effects on students?

Teacher Actions (Cause)	Student Practice (Effect)
	7. Look for and make use of structure. **DO STUDENTS:** • Look closely to determine a pattern or structure? • Utilize properties? • Decompose and recombine numbers and expressions?

Teacher Actions (Cause)	Student Practice (Effect)
	8. Look for and express regularity in repeated reasoning. **DO STUDENTS:** • Notice if calculations are repeated, and look both for general methods and for shortcuts? • Maintain oversight of the process, while attending to the details? • Continually evaluate the reasonableness of their intermediate result?

On	Off

On	Off

1	2	3
4	5	6
7	8	9

$$\frac{1}{5}$$

$$\frac{9}{10}$$

$$\frac{3}{3}$$

$$\frac{5}{8}$$

$$\frac{1}{6}$$

$$\frac{3}{8}$$

$$\frac{3}{5}$$

$$\frac{1}{8}$$

$\frac{7}{12}$	$\frac{4}{5}$
$\frac{7}{8}$	$\frac{1}{10}$

$\frac{4}{10}$	$\frac{2}{4}$
$\frac{11}{12}$	$\frac{5}{6}$

$$\frac{0}{3}$$

$$\frac{\quad}{\quad}$$

$$\frac{\quad}{\quad}$$

$$\frac{\quad}{\quad}$$

Time to Reflect

Summary

Cloze Reading Activity

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**

Mathematically ___________ students start by ____________ to themselves the meaning of a ___________ and looking for ___________ points to its solution. They ___________ givens, constraints, relationships and goals. They make ___________ about the form and meaning of the solution and plan a solution ___________ rather than simply jumping into a solution attempt.

3. **Construct viable arguments and critique the reasoning of others.**

Mathematically ___________ students understand and ____________ stated assumptions, definitions, and previously established results in constructing ___________. They make conjectures and build a logical progression of ___________ to explore the ___________ of their conjectures.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models,

they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

What would you like for the presenters to know?

1. What have you learned that would improve your instruction or student achievement?

2. What would you change?

3. What would you like more of?

The Use of Number Lines in CCSS

The number line first appears in the Measurement and Data domain in second grade where students are expected to represent numbers as lengths on the number line as well as sums and differences on a number line. Number line references are numerous in grade three, particularly in the Number and Operations domain, but in Measurement and Data as well. Reliance on the number line continues in grade four; references are seen through grade eight as well as in the high school Statistics and Probability domain. Not only does use of the number line persist across grade levels, but also across domains.

- The number line serves as a visual /physical model to represent the counting numbers and constitutes an effective tool to develop estimation techniques, as well as a helping instrument when solving word problems.

- The number line constitutes a unifying and coherent representation for the different sets of numbers which the other models cannot do.

- The number line is an appropriate model to make sense of each set of numbers as an expansion of other numbers and to build the operations in a coherent mathematical way.

- The number line enables one to present fractions as numbers and to explore the notion of equivalent fractions in a meaningful way.

- The number line, in some way, looks like a ruler, fostering the use of the metric system and the decimal numbers.

Share last - upper grades:

- The number line fosters the discovery of the density property of rational numbers.
- The number line provides an opportunity to consider numbers that are not fractions and consider the existence of irrational numbers.